FORMULE TRIGONOMETRICHE

(V. Colagrande)

FORMULE DI ADDIZIONE E SOTTRAZIONE

$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta$ $\qquad$ $\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta - \cos\alpha\operatorname{sen}\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$ $\qquad$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\operatorname{tg}\beta} \qquad \operatorname{tg}(\alpha-\beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha\operatorname{tg}\beta}$$

$$\operatorname{cotg}(\alpha+\beta) = \frac{\operatorname{cotg}\alpha\operatorname{cotg}\beta - 1}{\operatorname{cotg}\beta + \operatorname{cotg}\alpha} \qquad \operatorname{cotg}(\alpha-\beta) = \frac{\operatorname{cotg}\alpha\operatorname{cotg}\beta + 1}{\operatorname{cotg}\beta - \operatorname{cotg}\alpha}$$

FORMULE DI DUPLICAZIONE

$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\operatorname{sen}^2\alpha$

$$\operatorname{tg}2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha} \qquad \operatorname{cotg}2\alpha = \frac{\operatorname{cotg}^2\alpha - 1}{2\operatorname{cotg}\alpha}$$

FORMULE DI PROSTAFERESI

posto $\alpha+\beta = p$ e $\alpha-\beta = q$

$$\operatorname{sen}p + \operatorname{sen}q = 2\operatorname{sen}\frac{p+q}{2}\cos\frac{p-q}{2} \qquad \operatorname{sen}p - \operatorname{sen}q = 2\cos\frac{p+q}{2}\operatorname{sen}\frac{p-q}{2}$$

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2} \qquad \cos p + \cos q = -2\operatorname{sen}\frac{p+q}{2}\operatorname{sen}\frac{p-q}{2}$$

FORMULE DI WERNER

$$\operatorname{sen}\alpha\operatorname{sen}\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\operatorname{sen}\alpha\cos\beta = \frac{1}{2}\left[\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

FORMULE DI BISEZIONE

$$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad \operatorname{cotg}\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$$

FORMULE PARAMETRICHE

per $\alpha \neq (2k+1)\pi$ e posto $t = \operatorname{tg}\frac{\alpha}{2}$

$$\operatorname{sen}\alpha = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-t^2}{1+t^2} \qquad \operatorname{tg}\alpha = \frac{2t}{1-t^2}$$

FUNZIONI GONIOMETRICHE ESPRESSE MEDIANTE UNA SOLA DI ESSE

$$\operatorname{sen}\alpha = \pm\sqrt{1-\cos^2\alpha} = \pm\frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \pm\frac{1}{\sqrt{1+\operatorname{cotg}^2\alpha}}$$

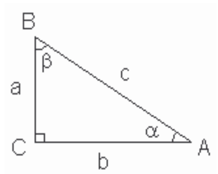
$$\cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha} = \pm\frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \pm\frac{\operatorname{cotg}\alpha}{\sqrt{1+\operatorname{cotg}^2\alpha}}$$

$$\operatorname{tg}\alpha = \pm\frac{\operatorname{sen}\alpha}{\sqrt{1-\operatorname{sen}^2\alpha}} = \pm\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha} = \frac{1}{\operatorname{cotg}\alpha}$$

$$\operatorname{cotg}\alpha = \pm\frac{\sqrt{1-\operatorname{sen}^2\alpha}}{\operatorname{sen}\alpha} = \pm\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{1}{\operatorname{tg}\alpha}$$

RISOLUZIONE DI TRIANGOLI RETTANGOLI

In un triangolo ABC, retto in C, avente a e b per cateti e c come ipotenusa, con α angolo opposto ad a e β angolo opposto a b, si ha:



IPOTENUSA: $c = \dfrac{a}{\operatorname{sen}\alpha} = \dfrac{a}{\cos\beta} = \dfrac{b}{\operatorname{sen}\beta} = \dfrac{b}{\cos\alpha}$

CATETI: $a = c\cdot\operatorname{sen}\alpha = c\cdot\cos\beta$

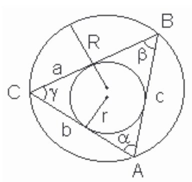
$b = c\cdot\operatorname{sen}\beta = c\cdot\cos\alpha$

$a = b\cdot\operatorname{tg}\alpha = b\cdot\operatorname{cotg}\beta$

$b = a\cdot\operatorname{tg}\beta = a\cdot\operatorname{cotg}\alpha$

RISOLUZIONE DI TRIANGOLI QUALUNQUE

Considerato un triangolo ABC, siano a, b e c i suoi lati e α, β e γ i rispettivi angoli opposti; siano R il raggio della circonferenza circoscritta al triangolo e r il raggio della circonferenza inscritta; siano S l'area e p il semiperimetro del triangolo.



1) **Teorema dei seni:** $\dfrac{a}{\operatorname{sen}\alpha} = \dfrac{b}{\operatorname{sen}\beta} = \dfrac{c}{\operatorname{sen}\gamma}$.

2) **Teorema della corda:** $\dfrac{a}{\operatorname{sen}\alpha} = \dfrac{b}{\operatorname{sen}\beta} = \dfrac{c}{\operatorname{sen}\gamma} = 2R$.

3) **Teorema di Carnot:** $a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha$; $b^2 = a^2 + c^2 - 2ac\cdot\cos\beta$;

$c^2 = a^2 + b^2 - 2ab\cdot\cos\gamma$.

4) **Area:** $S = \dfrac{1}{2}ab\cdot\operatorname{sen}\gamma = \dfrac{1}{2}bc\cdot\operatorname{sen}\alpha = \dfrac{1}{2}ac\cdot\operatorname{sen}\beta$; $S = \sqrt{p(p-a)(p-b)(p-c)}$ (formula di Erone).

5) **Raggi della circonferenza inscritta (r) e circoscritta (R):** $r = \dfrac{S}{p}$; $R = \dfrac{abc}{4S}$.

ANGOLI ASSOCIATI

funzione	Angolo							
	α	$\pi/2-\alpha$	$\pi/2+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$3/2\,\pi-\alpha$	$3/2\,\pi+\alpha$	$-\alpha$
cos	$\cos\alpha$	$\operatorname{sen}\alpha$	$-\operatorname{sen}\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\operatorname{sen}\alpha$	$\operatorname{sen}\alpha$	$\cos\alpha$
sen	$\operatorname{sen}\alpha$	$\cos\alpha$	$\cos\alpha$	$\operatorname{sen}\alpha$	$-\operatorname{sen}\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\operatorname{sen}\alpha$
tg	$\operatorname{tg}\alpha$	$\operatorname{cotg}\alpha$	$-\operatorname{cotg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$	$\operatorname{cotg}\alpha$	$-\operatorname{cotg}\alpha$	$-\operatorname{tg}\alpha$
cotg	$\operatorname{cotg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{cotg}\alpha$	$\operatorname{cotg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{cotg}\alpha$